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ABSTRACT

The focus of this paper is to further understanding of the distribution and the effects of an expanded conception of opportunity to learn on student mathematics achievement. In addition to descriptive statistics, a set of two-level hierarchical linear models was employed to analyze a subset of the restricted-use National Education Longitudinal Study of 1988 database. The results revealed that on different scales, various kinds of opportunities to learn mathematics are associated with student mathematics achievement, and opportunities are unequally distributed among different categories of schools. Four implications for educational policymaking are provided. They are: (1) the need to recruit, retrain, and retain teachers with adequate mathematical knowledge; (2) to encourage high content and level of instruction (including a high level of instruction, broad coverage, and an optimum amount of homework, which should be meaningful rather than merely busywork); (3) to provide more advanced mathematics courses, including replacing general mathematics with more advanced courses for the majority of the student body without decreasing the content; and (4) to increase learning opportunities in disadvantaged areas. Contains 85 references and 4 data tables. (Author/PVD)

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**THE DISTRIBUTION AND THE EFFECTS OF
OPPORTUNITY TO LEARN ON MATHEMATICS ACHIEVEMENT**

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Abstract:

The focus of this article is to further our understanding of the distribution and the effects of an expanded conception of opportunity to learn on student mathematics achievement. In addition to descriptive statistics, a set of two-level hierarchical linear models was employed to analyze a subset of the restricted use National Education Longitudinal Study of 1988 data base. The results revealed that, at different scale, various kinds of opportunity to learn mathematics do associate with student mathematics achievement and, unfortunately, opportunities are unequally distributed among different categories of schools. Four implications for educational policymaking are provided. They are: The need to recruit, retrain and retain teachers with adequate mathematical knowledge, to encourage high content and level of instruction (including high level of instruction, coverage, and appropriate amount of homework), to provide more advanced mathematics courses, and to increase opportunity in disadvantaged areas.

THE DISTRIBUTION AND THE EFFECTS OF OPPORTUNITY TO LEARN ON MATHEMATICS ACHIEVEMENT

Shin-Jiann Gau

Opportunity to learn (OTL) is a concept with a history of more than 30 years. It has been discussed in models of school learning (Carroll, 1963; Cooley & Lohnes, 1976; Harnischfeger & Wiley, 1976). The OTL concept was also introduced as a means to ensure the validity and comparability of cross-national comparisons in the First International Mathematics Survey in the early 1960s conducted by the International Association for the Evaluation of Educational Achievement. Such an important technical concept served as an explanatory variable in interpreting student mathematics achievement (McDonnell, 1995; Schmidt, Wolfe, & Kifer, 1993). The purpose was to take into consideration the curricular differences and the discrepancies in content coverage in comparing student mathematics achievement across different national systems. OTL was a measure of "whether or not the students have had an opportunity to study a particular topic or learn how to solve a particular type of problem presented by the test." (Husen, 1967, p. 162) It was broadly considered as a surrogate for national curriculum for a participating country (Schmidt & McKnight, 1995).

The OTL concept was refined in the Second International Mathematics Study (SIMS), conducted between 1976 and 1982. The implemented curriculum or implemented coverage was termed OTL (Travers, Garden, & Rosier, 1989; Westbury & Wolfe, 1989). Recently, researchers and others were expanding the concept of OTL beyond SIMS's primary focus on topic coverage. Research on determinants of students' achievement suggested that OTL should be defined not just by curriculum coverage but also by how that content was presented and who presented it (McDonnell, 1995; Shavelson, McDonnell, & Oakes, 1989; Wiley & Yoon, 1995). The Third International Mathematics and Science Study has employed "educational opportunity" to replace the traditional OTL (Schmidt & McKnight, 1995).

Empirical Relationships Between Previously Operationalized OTL Concepts and Students' Achievement

Empirical relationships between OTL and students' achievement have been established by early studies (e.g., Inkeles, 1977; Schmidt, 1983). In the past, the concept of OTL was often operationalized very narrowly as whether particular tested items were taught beforehand to students who took the test (e.g., Husen, 1967; Leinhardt & Seewald, 1981). That is, teachers' reports of content coverage was the sole indicator of OTL. There have been criticisms that it is "too bound to the form of specific items and more representative of teachers' judgements of items rather than content categories of which the item is an example" (Schmidt & McKnight, 1995, p. 345), and that it is "relatively imprecise

descriptive measures and should therefore be used cautiously" (Garden & Robitaille, 1989, p. 9) since teachers might misinterpret the items.

Efforts have been made by researchers to broaden the operational definition of OTL (e.g., Gau, 1996; MacIver & Epstein, 1995; MacIver, Reuman & Main, 1995; Muthen et al., 1995; Porter, 1993; F. I. Stevens, 1993, 1996). However, the results have not been that compelling. For example, MacIver and Epstein (1995) included eighth grade "algebra course content" and "teaching-for-understanding instruction" as OTL in their two studies. In their first study, the control variables [prior performance, socioeconomic status (SES), minority status, gender, and ability group level] explained 36.8% of the variance in student mathematics achievement at the student-level for public schools. After adding the course content variable (high-content, medium-content, and low-content course), the model accounted for only 38.7% of the variance in mathematics achievement at the student-level.¹

In MacIver and Epstein's (1995) second study, they used principals' report of classroom practice as the indicator for teaching for understanding in their schools' average and mixed-ability classes. Their model accounted for 64.3% of the variance in achievement at the school-level and only 6.6% at the student-level.

Muthen et al. (1995) employed student-reported class type as the sole indicator of the OTL for grade eight (algebra or prealgebra) and twelve (algebra, calculus, geometry, and trigonometry), and studied the effects of OTL and control variables altogether on student mathematics achievement. The figures of R square found in their study were between .39 to .44 (p. 393, 394, 396, and 397).

As above mentioned studies demonstrate, the proportion of variance explained at the student-level is not particularly high given the definitions of OTL in those studies. Although about 39% to 44% of the variance was explained, their full models included both control and OTL variables. The OTL concept explained only 1.9% of the variance at the student-level in addition to the control variables in MacIver and Epstein's (1995) study. Thus, previously operationalized definitions of OTL were not broad enough to account for a large proportion of the variance in student mathematics achievement. Therefore, there is room for improvement in studying the effects of OTL on students' achievement. Efforts are still needed to expand previous definitions of OTL, as the OTL concept is still at its "emerging idea stage" and "neither the what nor the why" has been finalized (Porter, 1995). Thus, finding the best means of measuring OTL remains a matter of concern for investigators even today (Burstein, 1993).

An Expanded OTL Concept

This study is aimed to further our understanding of the distribution and the net effects of a broadened conception of OTL on students' mathematics achievement. A school is an organization that exists to provide students with opportunity to learn. But OTL is not equally available at different schools, and within any one school, is not equally

available to all students (MacIver & Epstein, 1995; MacIver, Reuman, & Main, 1995; Sorensen, 1984). An investigation into the distribution of opportunity to learn mathematics across different categories of schools provides a fundamental understanding of the kinds of schools in which students' OTL is denied or restricted. Furthermore, after statistically controlling for effects derived from control variables, this study attempts to estimate the net, partial effect of an expanded OTL on student mathematics achievement and to determine if the expanded OTL explains more of the variance than previous studies did. By studying potentially "alterable" or "tractable" aspects of schooling, this study offers information on facilitating the understanding of how to create more effective and equitable schools.

What is OTL? The answer is different from one study to another. In this study, Opportunity to Learn is the conditions that may benefit students' mathematics learning and achievement, provided for students by the educational system. Operationally, the expanded OTL concept includes three constructs: teachers' mathematical knowledge, content and level of instruction, and school mathematical resources. Based on the literature in the field of mathematics education and relevant areas, Figure 1 depicts the major constructs in a conceptual model of opportunity to learn mathematics (for detail, see Gau, 1996). As argued by Gau (1996), these constructs are fundamental to OTL and necessary to explain the variation in student mathematics achievement.

<<Insert FIGURE 1 about here>>

Teachers' Mathematical Knowledge. Teachers' mathematical knowledge may influence the quality of instruction and hence the kind and quality of the opportunities that students receive. These opportunities in turn may have various effects on student mathematics achievement. Teachers' mathematical knowledge gained from preservice education (e.g., indicated by a mathematics degree) and professional development activities are important to what is done in classrooms and ultimately to what students will learn (Darling-Hammond, Wise, & Klein, 1995; Eisenberg, 1977; Fennema & Franke, 1992; Monk, 1994; Sternberg & Horvath, 1995; Thompson, 1992; Wiley & Yoon, 1995).

Content and Level of Instruction. The content and level of instruction to which students are exposed may affect their achievement (Reyes & Stanic, 1988). This construct includes high achievement group,² content (textbook/workbook) coverage, and content exposure (such as instructional time and amount of homework) (Camburn, 1996; Cooper, 1994; McDonnell, Burstein, Ormseth, Catterall, & Moody, 1990; Mullens & Bobbitt, 1996; Oakes, 1985; Robitaille & Travers, 1992; Schmidt & Burstein, 1993; Secada, 1992; Slavin, 1990).

School Mathematical Resources. A school's mathematical resources may influence the kind of classroom learning and instruction as well as the existence of extra curricular opportunity. The proportion of advanced mathematics courses (e.g., algebra classes at eighth grade), and the availability of instructional resources (such as school calculators), and

extra curricular opportunity (e.g., mathematics club) comprise this construct (Dossey, Mullis, Lindquist & Chambers, 1988; Jones, Davenport, Bryson, Bekhuis, & Zwick, 1986; Moore & Smith, 1987; Porter, 1995; Ralph, Keller, & Crouse, 1994; Secada, 1992).

In addition to the three above mentioned OTL-related constructs, student characteristics (gender, race, SES, and prior performance) and school characteristics (school sector, minority concentration, community type, and school average student SES) may influence what is learned from the opportunities provided. They are the control variables in this study.

Data and Analytic Methods

To explore the distribution and the effects of the OTL on students' mathematics achievement, this study drew raw data from the base year (1988, eighth grade) wave data files of the restricted use version of the second follow-up "National Educational Longitudinal Study of 1988" (NELS:88) data base. Conducted by the National Center for Education Statistics (NCES) at the U.S. Department of Education, the NELS:88 longitudinally collects the educational experiences and accomplishments of a nationally representative sample youth (NCES, 1994). For this study, 9,702 students located in 446 schools are selected as the samples for analysis. Mathematics teachers associated with the selected students are also included in order to provide necessary information. The students are selected on two conditions. First, a student has completed the survey and has a mathematics standardized score. Second, the student's mathematics teacher and school administrator have filed out their respective questionnaires in a school with at least 11 participating students.

Education is a multilevel, complex, highly contextualized system (Shavelson & Webb, 1995). So are the educational data obtained from the system. Its structures are often hierarchical, i.e., students are nested within schools (Bryk & Raudenbush, 1992; Burstein, 1980; de Leeuw, 1992; Gau & Wu, in press; Goldstein, 1995; Seltzer, 1995). It is important for a study to capture the complexity in a meaningful way, not to eschew it (Burstein, 1980; Shavelson & Webb, 1995).

The hierarchical linear modeling (HLM) technique allows a study to formulate a multilevel model that, in a joint analysis, estimates effects occurring at each of the levels and assesses the amount of variation explained at every level. Applying HLM to analyze student mathematics achievement, this study takes the hierarchically structured relationship into account to the extent that the NELS:88 data allows. Two-stage regression procedures are employed to examine the joint contribution of OTL at both student and school levels to student mathematics achievement. Specifically, in a two-level HLM, the Level-1 unit is students (student-level OTL variables, the associated teachers' and parents' information, and control variables); and the Level-2 unit is school (school-level OTL variables, and control variables). That is, this study estimates effects occurring within-school (student-level effects) at Level-1, and those occurring between-schools (school-level effects) at Level-2 respectively. To investigate the impact the OTL has on student

mathematics achievement, this study statistically controls for those control variables and focus on the partial, net effects of OTL. It is of interest to see if the partial effect of OTL is significant while holding other factors constant.

The specific equation at student- and school-level for this study are as follows:

$$Y_{ij} = \beta_{0j} + \beta_{1j}(\text{Gender})_{ij} + \beta_{2j}(\text{Race})_{ij} + \beta_{3j}(\text{SES})_{ij} \quad [\text{Student-level}]$$

$$+ \beta_{4j}(\text{Prior Performance})_{ij}$$

$$+ \beta_{5j}(\text{Teachers' Mathematics Degree})_{ij}$$

$$+ \beta_{6j}(\text{Teachers' Professional Development})_{ij}$$

$$+ \beta_{7j}(\text{Higher Achievement Group})_{ij}$$

$$+ \beta_{8j}(\text{Textbook Coverage})_{ij}$$

$$+ \beta_{9j}(\text{Instructional Time})_{ij}$$

$$+ \beta_{10j}(\text{Amount of Homework})_{ij} + r_{ij}$$

$$\beta_{0j} = \gamma_{00} + \gamma_{01}(\text{Catholic Sector})_j \quad [\text{School-level}]$$

$$+ \gamma_{02}(\text{Other Private Sector})_j$$

$$+ \gamma_{03}(\text{Minority Concentration})_j$$

$$+ \gamma_{04}(\text{Suburban School})_j + \gamma_{05}(\text{Rural School})_j$$

$$+ \gamma_{06}(\text{School Average Student SES})_j$$

$$+ \gamma_{07}(\text{Proportion of Algebra Classes})_j$$

$$+ \gamma_{08}(\text{Access to School Calculators})_j$$

$$+ \gamma_{09}(\text{Mathematics Club})_j + u_{0j}$$

Descriptive Statistic Results: The Distribution of OTL Across Schools

The next three sections present results of the data analysis and discussions of the findings. The descriptive statistics results offer delineation of the distribution of opportunity to learn mathematics across schools. The HLM analysis results provide the effects of OTL on student mathematics achievement.

This section provides descriptions of OTL variables against school-level control variables.³ These descriptive statistics are presented since they are able to provide a fundamental understanding of the distribution of opportunity to learn mathematics across schools. These descriptions are intended to illustrate the kinds of schools in which students' opportunity to learn mathematics are denied or restricted.

The Distribution of OTL Across School Sectors by School Average Student SES

Table 1 presents the distribution of OTL across school sectors by school average student SES. Note that these tables are intended to provide a descriptive profile of tendencies with no implication of statistical significance since figures in these tables are descriptive in nature.

<<Insert TABLE 1 about here>>

As a whole, students in non-Catholic religious and nonsectarian private sector outperform those in the Catholic sector [55.3 vs. 51.7, with an effect size (E.S.) of .721, which is quite large; that is, the difference between the means is about $(72/100)\sigma$]. These Catholic school students in turn have a slightly higher mean mathematics achievement score than those in the public sector (49.3, E.S. = .481). Eighth graders in public schools have the highest opportunity to receive mathematics instruction from teachers who possess a mathematics degree (41.5% vs. 17.5% and 30.2%). These public school students also have more opportunity than others to use school-owned calculators (40.5% vs. 24.9% and 28.3%) and to attend a mathematics club (29.4% vs. 16.9% and 17.7%).

On average, mathematics teachers in Catholic schools and other private schools "cover" most of the eighth grade mathematics textbook/workbook (90.1% and 89.5% respectively). Students in Catholic schools receive the highest amount of mathematics homework per week (169.4 minutes vs. 150.7 of public schools and 138.9 of other private schools). However, less of the eighth grade student body (about 14.5%) in Catholic schools attend a class whose achievement level is classified by their teachers as higher than average. Catholic schools provide their students with less opportunity to attend a mathematics club (16.9%) than the other two sectors (29.4% and 17.7%).

About half of the students in non-Catholic religious and nonsectarian private schools have attended classes categorized by their teacher as higher than average (49.9% vs. 22.6% and 14.5%). More than half of the eighth grade student body in these schools have attended algebra classes (54.0%), compared to approximately only one third in public and Catholic schools (31.0% and 37.6%).

Schools with high average student SES have higher mean mathematics achievement score than that of schools with middle average student SES (54.0 vs. 51.1, E.S. = .581), which in turn is higher than that of schools with low average student SES (47.1, E.S. = .802). Students in schools with high average student SES have more chance than others to receive mathematics instruction from teachers with a mathematics degree (40.5% vs. 29.5% and 34.5%). The highest proportion of this student body has attended classes categorized by their teachers as higher than average (33.9% vs. 18.6% and 18.7%). About half (49.2%) of this student body have attended algebra classes, which is much higher than those who attend school with middle and low average student SES (30.1% and 27.0%).

As Table 1 shows, the public sector has a higher proportion of schools with low or middle average student SES than the other two private sectors. However, no matter what the sector, the higher the school average student SES, the higher the mean school mathematics achievement. There are relatively small differences in achievement across the three sectors within each SES level. The following sections offer detailed descriptions of each sector.

Non-Catholic Religious and Nonsectarian Private Schools. All the non-Catholic religious or nonsectarian private schools have high school average student SES, except for one with middle average student SES as Table 1 shows. Students in these schools has the highest mean mathematics achievement (but only 1.7 points above the mean of public schools with high average student SES, E.S. = .341). More than half (55.4%) of their students has attended classes with achievement levels that are classified by their teachers as higher than average. It is much higher than the other two sectors. More students in these schools have attended an algebra class at least once a week (about 60%) than in high average student SES public (41.8%) or Catholic (47.9%) schools. On the other hand, they have less opportunity than any of their public school counterparts to have access to a school calculator and to attend a mathematics club. However, one could surmise that if their parents could afford to send them to a high average student SES private school, then they are likely to have had a personal calculator provided for them. Interestingly, considerably fewer teachers (33.5%) in these high average student SES private schools have a mathematics degree than their counterparts in high average student SES public schools (61.2%).

Public Schools. The differences in the distribution of OTL are especially substantial across the school average student SES in public sector. Public schools with low or middle average student SES tend to provide fewer mathematical opportunities to their eighth graders than their high average student SES counterparts. In terms of teachers' mathematical knowledge, a relatively small proportion of the eighth grade teachers in public schools with low or middle average student SES possess a mathematics degree (36.7% and 40.6% vs. 61.2%). These teachers also have spent slightly fewer hours in professional development activities than their counterparts in high average student SES schools (11.8 hours and 9.7 hours vs. 14.1 hours).

As to the "content and level of instruction," the low or middle average student SES public schools seem to provide less opportunity to their eighth graders than their high average student SES counterparts. A smaller proportion of their eighth grade student body (19.3% and 21.9% vs. 36.4%) have attend a mathematics class categorized as a "higher level" class. The percentage of textbook/workbook coverage is also slightly lower for these students, although they receive somewhat longer instructional time per week and students in low average student SES schools have slightly more homework than students in middle or high average student SES schools. Also, low or middle average student SES public schools offer fewer learning opportunities to their students in terms of advanced mathematics courses (algebra) (26.5% and 33.4% vs. 41.8%), instructional resources (pocket or hand-held calculators) (39.1% and 36.2% vs. 55.1%), and a mathematics club (21.3% and 33.4% vs. 49.8%) than their high average student SES counterparts.

These descriptive statistics imply that public schools with low or middle average student SES are deficient, compared to their high average student SES counterparts, in providing mathematical OTL to their students. Although students in these schools receive somewhat longer

instructional time per week and have slightly more homework than students in other schools, given the fact that most classes are not higher level, the additional instructional time and homework is unlikely to contribute to student mathematics learning in advanced areas.

Catholic Schools. The distribution of OTL among Catholic Schools is more mixed than in public schools, i.e., there is not a clear trend across Catholic schools by school average student SES as there is in public schools. However, Catholic schools with high average student SES have a higher proportion of teachers possessing a mathematics degree and have a higher proportion of students who are assigned to a higher than average class and who have attended algebra classes at least once a week as compared with low or middle average student SES schools (28.0% vs. .0% and 8.6%; 16.6% vs. 10.5% and 12.8%; and 47.9% vs. 33.7% and 25.4% respectively). Mathematics classes in these high average student SES schools meet slightly longer than classes in low or middle average student SES schools each week. Teachers in these high average student SES schools assign the highest amount of weekly mathematics homework to their students (184.2 minutes) among the three sectors. On the other hand, students in these high average student SES schools have less opportunity than their counterparts in low or middle schools to use school-owned calculators (21.4% vs. 47.6% and 23.7%). However, this finding may be because they are required to have their own calculator or their parents are able to provide personal calculators for them. As to other opportunity (teachers' participation in professional development activities, textbook coverage, and mathematics club), the results are mixed.

Section Conclusions. Students attending schools with low or middle average student SES are usually in a doubling disadvantaged situation. Most of them grow up in low SES families, which have long been regarded as the primary determinant of variations in performance (Bridge, Judd, & Moock, 1979; Hanushek, 1994; Murnane, 1975; Secada, 1992; White, 1982). Such a disadvantage is further worsened when they attend low or middle average student SES schools, since these schools tend to provide their students with less opportunity to learn mathematics than do their affluent counterparts. In short, schools with high average student SES tend to provide more mathematical learning opportunity to their students than their middle or low counterparts. The differences are especially substantial among public schools.

Among these three school sectors, public schools have the highest proportion of teachers (41.5%) possessing a mathematics degree. These schools also had the highest proportion of the eighth grade student body with the opportunity to use school-owned pocket or hand-held calculators and to attend a mathematics club. Teachers in Catholic schools, except for low average student SES schools, assigned students the highest amount of weekly homework. Non-Catholic religious or nonsectarian private schools had the highest proportion of students attending higher than average classes and algebra classes (55.4% and 59.9% respectively). However, it must be kept in mind that all but one of these schools had high average student SES.

The Distribution of OTL Across School Location by Minority Concentration

Table 2 presents the distribution of OTL among urbanicity of school location by minority concentration of eighth graders. Overall, the mathematics achievement among urbanicity of school location is similar. However, it varies across different levels of concentration of eighth grade minority students--the higher the concentration, the lower the mean mathematics achievement score (43.7, 48.1, and 51.5 respectively, E.S. = -.882 and -.681). A higher proportion of students in urban and suburban schools than in rural schools have attended algebra classes (39.73% and 39.08% vs. 27.04%). Urban schools provide their students more opportunity to join a mathematics club than do suburban and rural schools (34.80% vs. 22.79% and 21.63%). Students in suburban schools are assigned the highest amount of homework per week, compared with urban and rural schools (165.2 minutes vs. 150.8 and 145.5 minutes.)

<<Insert TABLE 2 about here>>

High minority concentration schools have the lowest percentage of students attending classes categorized as higher than average (8.87% vs. 27.91% and 24.42%). However, these students receive the highest amount of homework per week (171.2 minutes vs. 165.4 and 150.5 minutes). Also, their teachers have attended professional development activities more often than teachers in the other two categories of schools (17 hours vs. 11 and 11 hours).

Predictably, suburban and rural areas have disproportionally low percentages of schools with middle or high minority concentration. However, the pattern of average school mathematics achievement is similar for urban, suburban, and rural schools--the higher the eighth grade student minority concentration, the lower the average school mathematics performance.

The distribution of OTL is mixed across Table 2. However, across the three urbanicity locations, high minority concentration schools tend to provide their eighth graders less opportunity than their middle or low minority counterparts. For example, a substantially smaller proportion of the eighth grade students in high minority concentration schools than low or middle ones have the opportunity to attend classes categorized as high level by their teachers. The figures are especially low for suburban and rural schools (7.0% and 5.2% respectively). Urban and especially rural schools with a high minority concentration tend to provide their eighth grade students less opportunity than their low or middle counterparts to take an advanced mathematical course (algebra) (37.4% vs. 40.9% and 38.3%; and 13.6% vs. 26.9% and 48.7%) and their classes also "cover" less of the textbook/workbook than in the low or middle minority concentration schools (82.3% vs. 87.4% and 88.1%; and 77.1% vs. 85.7% and 88.2%). In other words, high achievement group in terms of the levels of advancement of course seem to be more common in high minority schools than in low or middle ones, especially in urban and rural areas.

These high minority concentration schools, however, do provide their students with more opportunity in certain areas (e.g., teachers' participation in professional development activities, instructional time, weekly mathematics homework, and mathematics club) than do low or middle minority concentration schools. The finding that these schools turn out to have low average school mathematics achievement suggests that some opportunities may be more important, and thus have stronger association with achievement, than others. This speculation is investigated in the next section.

HLM Analysis Results:

The Effects of OTL on Student Mathematics Achievement

This section presents and discusses the results of the HLM analyses. As mentioned previously, data are analyzed within the framework of a set of two-level hierarchical linear models using the computer program HLM. To facilitate the discussion, the results are divided into the student- and school-level. Table 3 summarizes the results of the analyses at the student-level (within-school level), while Table 4 presents the school-level (between-school level) findings. Note that Table 3 has two sub-models and Table 4 includes three sub-models. In the proposed model of both Tables, all the variables mentioned in "An Expanded OTL Concept" section are included. On the contrary, two OTL variables, teachers' mathematical knowledge gained from professional development activities and instructional time, are excluded in the revised model of both Tables due to their counter commonsense correlation with student mathematics achievement.⁴ Table 5 includes an additional sub-model, "Without School SES Model," in which the school average student SES is eliminated in the HLM analysis (detailed in "School Mathematical Resources" section).

Teachers' Mathematical Knowledge

The finding in relation to teachers' mathematical knowledge is that after statistically controlling for eighth grade students' gender, race, SES, prior performance, other OTL variables, and school-level variables, student mathematics achievement is higher when the student's teacher possesses more mathematical knowledge. In the proposed model (see Table 3), the two measures about teachers' mathematical knowledge are both statistically significant, but in opposite directions.

<<Insert TABLE 3 about here>>

Mathematics Degree. After statistically controlling for eighth grade students' gender, race, SES, prior performance, other OTL variables, and school-level variables, mathematics achievement is higher when the student's teacher possessed a mathematics degree. The expected net teacher mathematics degree gap (β_{5j} , after controlling for other variables, the mean difference between the mathematics achievement of

students whose teacher do not possess a mathematics degree and those whose teacher do) is .278 point ($t = 5.857$). The effect size of the teacher mathematics degree is .040, which is quite small; that is, the difference between the means is about $(4/100)\sigma$. The teacher mathematics degree gap is increased from .278 to .362 point in the revised model (the E.S. is changed from .040 to .052). These results support the speculation that a mathematics degree is an indicator of teachers' knowledge related to the course they teach in school. However, the small latitude of effect size may be because some teachers with a mathematics degree have been purposefully assigned to classes with low achievement students in order to help those students.

Professional Development Activities. The expected differentiating effect of teachers' professional development activities ($\hat{\beta}_{6j}$) is -.054 ($t = -2.731$). The standardized $\hat{\beta}_{6j}$ ($S\hat{\beta}_{6j}$) is -.008, which is small and runs counter to the speculated direction. (The symbol " $S\hat{\beta}$ " is used to represent the expected standardized regression weight). That is, opposite to the expectation, the more time the student's teacher has spent in professional development activities, the lower the student mathematics achievement. Such a result is very different from common thinking about the improvement of teaching and learning, but it might be understandable in light of certain circumstances for encouraging teacher participation in staff development. For example, as a local school district's curriculum coordinator revealed in an interview conducted for another study, certain teachers have been "suggested" by him to attend particular staff development activities.⁵ These teachers were those whom he thought were in need of improvement. However, since the NELS:88 does not contain a measure to detect teachers' teaching effectiveness and thus it is difficult to disentangle which types of teachers participated in professional development activities.

Furthermore, Curriculum and Evaluation Standards for School Mathematics (1989) and Professional Standards for Teaching Mathematics (1991) were both published by the National Council of Teachers of Mathematics (NCTM) after the NELS:88's base year wave data collection in 1988. Professional development activities provided by school districts and other professional organizations prior to these two NCTM standards documents may not reflect more recent reform-oriented emphases (e.g., higher-order thinking, teaching for understanding). Thus, it might be problematic to use NELS:88 base year teachers' time spent in professional development activities as an indicator for the knowledge suggested by the recent mathematics reforms. A test of later waves (e.g., 1990 and 1992) of the NELS:88 data may result in different conclusions.

In short, for NELS:88 base year students, teachers' mathematics degree (undergraduate or/and graduate) is a positive indicator of students' opportunity to learn mathematics. On the other hand, teachers' time spent in professional development activities in mathematics is a negative

indicator of students' OTL. However, due to the arguments provided above, this variable is not included in the revised model.

Content and Level of Instruction

This study speculates that after statistically controlling for eighth grade students' gender, race, SES, prior performance, other OTL variables, and school-level variables, the higher the level of instruction, the higher the student's mathematics achievement. The results of the content and level of instruction analyses are mixed as well. Three of the four variables are statistically significant in a positive direction, while the other is significant in a negative direction.

High Achievement Group. After statistically controlling for eighth grade students' gender, race, SES, prior performance, other OTL variables, and school-level variables, student mathematics achievement is higher when the student attended a higher than average achievement level mathematics class. The expected net high achievement group gap ($\hat{\beta}_{7j}$, after controlling for other variables, the mean difference between the mathematics achievement of non-higher level class students and of higher level class students) is 8.758 ($t = 235.952$) in the proposed model and 8.752 in the revised model. The E.S. is 1.255 in proposed model and 1.254 in revised model. Both are the highest coefficients among all the variables specified in the current study.

The differentiation of the mathematics curriculum goes together with the practice of grouping. Thus, students in different groups are exposed to very different kinds of mathematics (Burks, 1994; Kifer, 1993; Schmidt et al., 1993; Sorensen, 1984; Wheelock, 1992). Also, higher-order thinking instructional approaches tend to be used more frequently in higher achievement groups (e.g., honors- and academic-track classes) than in other groups (e.g., general- or vocational-track classes) (MacIver & Epstein, 1995; Raudenbush, Rowan, & Cheong, 1993). Thus, although, the sorting is far from flawless (Guiton & Oakes, 1995; Kifer, 1993), the differentiated opportunities for learning mathematics become a reality for different groups.

Content Coverage. The empirical testing suggested that after statistically controlling for eighth grade students' gender, race, SES, prior performance, other OTL variables, and school-level variables, the higher the percentage of the textbook/workbook is covered by the student's teacher, the higher the student mathematics achievement. The expected net differentiating effect of content coverage ($\hat{\beta}_{8j}$) is .333 ($t = 19.223$) in the proposed model and .315 in the revised model. The $S\hat{\beta}_{8j}$ is .061 and .057 in respective models, although statistically significant, these effects are both small to a limited extent. This implies that students who attend a mathematics class with a higher rate of textbook/workbook coverage may have more opportunity to learn mathematics and therefore have slightly higher mathematics achievement than those who attend a class with lower content coverage. Generally, a student gains an

additional .315 point in the NELS:88 mathematics test battery for each 10% increase in coverage of the mathematics textbook/workbook.

Content Exposure: Instructional Time. The direction for the differentiating effect of instructional time is opposite to the proposed speculation. That is, the analysis shows that after statistically controlling for eighth grade students' gender, race, SES, prior performance, other OTL variables, and school-level variables, the longer the mathematics class regularly met per week, the lower the student mathematics achievement. The expected net effect ($\hat{\beta}_{9j}$) is $-.777$ ($t = -19.804$) point per each hour of weekly instructional time above the 2 hours base. The $S\hat{\beta}_{9j}$ is $-.079$, which is very small. Such a result runs counter to a commonsense understanding of mathematics learning.

Based on an analysis of the NELS:88 data set, all 1988 eighth graders virtually took a full year of mathematics. But they spent different amounts of time in learning mathematics. It is quite possible that those who learned mathematics slower than others and those who had low mathematics achievement took more time to make up their learning. Also, different kinds of instruction may have different effects. For example, it is unlikely that an extra minute of group time has the same effect on learning as a minute of tutoring time (Bloom, 1984). As a result, it is impossible to get clear estimates of the productivity of time unless the researcher has information not only on the allocated time, but also on how and over which type of activities it was used (Brown & Saxe, 1986). This variable is eliminated in the revised model as stated earlier.

Content Exposure: Amount of Mathematics Homework. As conjectured, after statistically controlling for eighth grade students' gender, race, SES, prior performance, other OTL variables, and school-level variables, the more weekly mathematics homework, the higher the student mathematics achievement. The expected net differentiating effect of weekly mathematics homework ($\hat{\beta}_{10j}$) is moderately positive relative to student mathematics achievement. Generally, an additional 10 minutes of homework in each week contributes an additional .146 point (in the proposed model and .140 in the revised model) to the students' score in the NELS:88 mathematics test battery. The $S\hat{\beta}_{10j}$ is .174 in proposed model and .167 in revised model. Both effects are small. The result supports the idea that doing homework, as one of the ways students spend some of their time outside of school, may enhance their in-school academic performance (Lapointe, Mead, & Askew, 1992).

In sum, except for instructional time, the higher the level of instruction, the higher the student's mathematics achievement. In terms of OTL, high achievement group, textbook/workbook coverage, and weekly homework contribute to students' mathematical learning and thus positively associate with their achievement. Instructional time, on the other hand, has the opposite direction. Further research should be devoted to disentangle such a counter commonsense result for the NELS:88 sample students.

As preceding analyses showed, among the student-level OTL variables, the variable of high achievement group has the strongest relationship with student mathematics achievement score (E.S. = 1.254).

The effect of the amount of homework per week ($S\hat{\beta}_{10j} = .167$) is second to the high achievement group. The $S\hat{\beta}$ of textbook/workbook coverage (.057) and the E.S. of teachers' mathematics degree (.040) are both smaller than those of high achievement group and weekly homework. It means that content coverage and weekly homework tend to have weaker association with student mathematics achievement scores than do the other two OTL variables.

School Mathematical Resources

In neither Level-1 proposed or revised models, the HLM analyses do not support the speculations that after statistically controlling for school sector, school minority concentration, community type, school average student SES⁶, and student-level OTL variables, the more the school mathematical resources, the higher the school mean student mathematics achievement (see Table 4). Such a result, however, does not necessarily mean that school mathematical resources have nothing to do with mean school mathematics achievement.

<<Insert TABLE 4 about here>>

Examining the control variables in Table 4, a reader will find that the school average student SES has explained a large proportion of the variance in the mean school mathematics achievement (see also Table 1). In addition to its social connotations, the school average student SES indicates the economic part of life as well. It is an indicator of the resources and financial support that a school may obtain from the surrounding community. Thus, a school with high average student SES often means that it is located in an affluent area and thus is likely to gain more resources from its community than schools in less affluent areas.⁷ Part of the resources may be in the form of instructional resources and therefore contribute to students' academic learning in general and mathematics learning in particular.

Therefore, school average student SES may cancel other control variables and may have masked the effects of school mathematical resources. To examine this possibility, an additional analysis is conducted in which the school average student SES is excluded while other variables are retained in the HLM model. The results differed dramatically (see "Without School SES Model" in Table 4).

One of the school mathematical resources, the percentage of students attending algebra classes, changes from statistically not significant to significant at $p < .01$ level. Its coefficient (γ_{07}) increased from -.005 (in proposed model or -.004 in revised model) to .025 (the standard error changes from .007 to .008, and t -ratio, from -.685 and -.596 to 3.177).

The standardized γ_{07} changes from $-.050$ in proposed model (and $-.041$ in revised model) to $.218$. The degree to which differences in the percentage of eighth grade students attending algebra classes becomes positive related to their school mean mathematics achievement. That is, a school that has an additional 10% of its eighth grade student body attending algebra classes at least once a week tends to have a mean mathematics achievement that is higher by a small effect size of $.088$ [or $.25$ ($.025 \times 10$) point in NELS:88 mathematics test battery].

The other two school mathematical resources (the availability of school calculators and a mathematics club) are also positively related to mean school mathematics achievement but at non-significant levels. As argued in the "Descriptive Statistics Results" section, this might be because students who attend schools with high mean mathematics achievement have their own personal calculator. Since there is no information on what kind of activities and content mathematics clubs offer, they might not contribute to mathematics test taking or to enhance students' knowledge on what is included in the NELS:88 mathematics test battery. These results mean that once the general school resources measure (indicated by school average student SES) is not entered into the model, at least one of the school mathematical resources defined in the expanded OTL definition (the percentage of students attending algebra classes) matters to some extent.

Among the control variables, the minority concentration remains statistically significant. Its coefficient increases from $-.014$ (or $-.016$ in revised model) to $-.049$ and its significance level moves from $p < .05$ level to $p < .001$ level (the standard error remains the same, $.006$). This means that a 10% increase of minority concentration in the eighth grade student body is related to a $.49$ ($-.049 \times 10$) point decrease in the school mean mathematics achievement. The minority concentration gap increases about three and half times (per each percentage) after the school average student SES is eliminated. Also, both "Catholic school" and "other private school" (non-Catholic religious or nonsectarian private schools) change from statistically nonsignificant to significant at $p < .05$ and $p < .001$ level respectively. That is, the mean school mathematics achievement of both private sectors are higher than that of their public counterparts. This might suggest that variables of minority concentration and school sector are confounded with the school average student SES. If this is the case, the result that school mean mathematics achievement is lower in higher minority concentration schools and in public schools might have something to do with the low average student SES environment, rather than the minority concentration or school sector itself.

Therefore, as the investigator suspected, due to a large proportion of the variance in school mean mathematics achievement being explained by school average student SES, other variables such as school mathematical resources are not significant. They are not significant in a statistical sense, but are not necessarily insignificant in a practical sense. The above discussion implies that when studying school mathematics resources, the

measurement of general school resources (e.g., school average student SES) should be excluded from the model.

Proportion of Variance Explained

Using the revised model, this section analyzes the proportion of variance explained by the constructs introduced in this study. Tables 5 and 6 present the HLM analysis results for each construct at the student- and school-level respectively. In both Tables, the top row offers the variance of each model, while the bottom row provides the proportion of reduction in variance or "variance explained" by each of the fitted model from the "fully unconditional model."

<<Insert TABLE 5 and TABLE 6 about here>>

At the student-level, the "control variables only" model accounts for 25.81% of the variance in student mathematics achievement. The teachers' mathematics knowledge construct explains only an additional .23% of the variance. The content and level of instruction, on the other hand, explains 14.11% of the variance in addition to the control variables. In total, the net, partial effect of student-level OTL variables constructed in this study explains 14.12% of the variance in student mathematics achievement after statistically controlling for other variables. The whole student-level model accounts for 39.93% of the variance.

At the school-level, the control variables explain most, 64.12%, of the variance in school mean mathematics achievement. The school mathematical resources, the school-level OTL construct, explains virtually no variance at all (only .24%) after statistically controlling for the school sector, school minority concentration, community type, and school average student SES.

In sum, the student-level OTL variables explain 14.12% of the variance in addition to the control variables. The school-level OTL variables account for virtually nothing since the control variables (especially the school average student SES) have explained a large proportion of the variance at this level. Overall, after statistically controlling for effects derived from control variables, the expanded OTL concept developed in this study explains a moderate amount (14.12%) of the variance in student mathematics achievement at the student-level and little if any variation at the school-level.

Implications for Educational Policy

Mathematics achievement has long been and remains a major focus of policy, research, and public concern. Unfortunately, again and again, the performance of U.S. cohorts in international studies has been unfavorable, and lagging behind that of other industrialized countries (see e.g., Lapointe et al., 1992; McKnight et al., 1987; NCES, 1995; Stevenson, Lee, & Stigler, 1986; Stevenson & Stigler, 1992).

Nevertheless, a number of authors have argued from a different perspective. Former Secretary of Education Lamar Alexander pointed out that "today's children seem to know about as much math and about as much science and read about as well as their parents did at that age 20 years ago" (U.S. Study shows, 1991). Others interpreted the results from National Assessment of Educational Progress as students' average performance being significantly higher in 1990 than 1978 (Beaton & Zwick, 1992; Mullis, Dossey, Foertsch, Jones, & Gentile, 1991). Or, "For the nation, there were statistically significant increases in average mathematics proficiency between 1990 and 1992...." (Mullis, Dossey, Owen, & Phillips, 1993, p. 1). Also, "American schools have never achieved more than they currently achieve...." (Bracey, 1991, p. 106, 110).

In the 1990s, there is a rhetorical shift from the view that public schools are suffering from declining student academic achievement toward the view that achievement levels are not what they need to be to meet the challenges of the coming decades. The question that "Are we good enough to stand up to worldwide competition?" (Kirst, 1993, p. 613) become a major concern among researchers and educational policymakers. This "revisionist" view become dominant within just a few years (Ralph et al., 1994).

Student mathematics performance become one of the major concerns of the "Goals 2000: Educate America Act." It specifies that "By the year 2000, all students will leave grades 4, 8, and 12 having demonstrated competency over challenging subject matter including ... mathematics, ..." and ambitiously challenges that "By the year 2000, United States students will be first in the world in mathematics and science achievement." Such a strong, growing policy concern and aspiration does not come out of a vacuum. The drive towards improving educational productivity seems always to be on the policy and research agenda (Monk, 1992; Odden, 1992).

Various kinds of opportunity to learn mathematics do associate with student mathematics achievement and, unfortunately, opportunity is unequally distributed among schools. These are the two major implications that the present study has for educational policymaking.

Based on the findings of this study, the following four recommendations are offered for educational policymakers trying to improve student mathematics achievement. Nevertheless, it is important to emphasize again that test scores and academic achievement are not the only outcome parents expect from children's schooling and that the mathematics test may not reflect what students seek from their education. Also, these implications are derived from correlational associations, not causal relationships, among certain variables that were found to be statistically associated with student mathematics achievement. Thus, the following recommendations are based not solely on the statistical findings, but on logical arguments that can be offered to make connections between various factors included in this study and broader educational policies and practices.

Recruit, Retrain and Retain Teachers With Adequate Mathematical Knowledge

Schools should provide all students with the opportunity to receive mathematical instruction from teachers with adequate mathematical knowledge. It might be considered that eighth grade mathematics teachers don't need a great deal of mathematical knowledge for their instruction. However the results of this study indicate that teachers' possession of a mathematics degree is positively associated with their eighth graders' mathematics achievement. Since "No one questions the idea that what a teacher knows is one of the most important influences on what is done in classrooms and ultimately on what students learn" (Fennema & Franke, 1992, p. 147), schools should provide students with teachers who have adequate mathematical knowledge.

Only 44.8% of the eighth graders' mathematics teachers possess a bachelor or/and graduate degree in mathematics. It is the responsibility of educational authorities to recruit qualified mathematics teachers into the teaching profession and retain them. For those teachers already in the profession, but without a mathematics degree, the educational system should encourage them to pursue graduate training in mathematics if they are going to continue to teach mathematics. The education system will then be able to provide all of their eighth graders with the opportunity to learn mathematics from a teacher with adequate mathematical knowledge.

Encourage High Content and Level of Instruction

High Level of Instruction. Schools should provide all students with the opportunity to receive the kind of mathematical instruction that has the qualities commonly shared by high track classes, for example, emphasizing teaching for understanding, higher-order thinking, high/appropriate expectations from their teachers, and challenging content. Despite recent calls to teach for understanding and higher-order thinking to all students, studies have found that higher-order thinking instructional approaches are much more frequently employed in high achievement groups than in other classes. In the present study, 26.8% of the eighth grade students are placed in a classroom identified as higher than average level and thus are more likely than other students to receive instruction that emphasized higher-order thinking.

Although the sorting is far from flawless, tracking is the reality of school life for most students. It is also impractical to place all students in high tracks. If tracking is retained, all students should have the opportunity to receive the same quality of instruction. Given that low-achieving high school students have been found capable of learning more than is typically demanded of them (Gamoran, Porter, Smithson, & White, 1996), eighth grade students are likely to be similarly able. No matter which groups students are in, they all should be provided with the opportunity to receive instruction that has the above mentioned characteristics commonly shared among high tracks. Teachers teaching lower track classrooms should be encouraged to provide their students with the kind of instruction that has these qualities.

Coverage. Schools should provide all students with the opportunity to receive mathematical instruction that covers the content of major concepts specified for the eighth grade. In the present study, 12.0% of the eighth graders' mathematics teachers cover less than 70% of the content of their textbook/workbook, while 15.1% cover 70-79%, 34.2% cover 80-89%, and 30.6% cover 90-99%. Only 8.1% of the students receive 100% content coverage. Therefore, a substantial proportion of the eighth grade student body does not have the opportunity to cover at least 90% of the textbook/workbook selected for their grade. Of course, the eighth grade mathematics curriculum does not necessarily equal the textbook/workbook selected for the grade. Sometimes, it might not be necessary to cover all the topics in a selected text in order to cover the curriculum.

Thus, when mathematics test scores are a major concern to educators and the public, it may become desirable to encourage eighth grade teachers to cover as much of the content of the selected textbook/workbook as possible, especially when tests cover a broad range of topics. Such a suggestion is supported by other empirical studies as well. For example, an international comparison study found that the more the textbook content is covered in a country, the higher the student mathematics achievement (Kifer & Burstein, 1993). It suggests that providing more content to more students will produce more gain in test scores--"the amount of learning increases as comprehensiveness increases" (Kifer & Burstein, 1993, p. 337).

Nevertheless, too often the press for coverage may result in only superficial coverage and a focus on learning facts rather than understanding concepts. Given the current promotion of higher-order thinking, teaching for understanding, and other forms of assessment of students' academic performance (e.g., authentic assessment, portfolio), it might be desirable to encourage high coverage of the major concepts in the curriculum instead of the textbook content.

Appropriate Amount of Homework. Schools should provide all students with the opportunity to receive an appropriate amount of homework assignments so that they can work further with the content taught in class in out-of-school time. In the present study, about one third of the students regularly receive mathematics homework assignments requiring less than one hour per week, which is less than 12 minutes of mathematics homework per day. Fewer than half (46.3%) of the students are assigned at least 30 minutes of mathematics homework per day (i.e., at least 150 minutes weekly). Only 16.7% of the students received at least 45 minutes daily mathematics homework (i.e., at least 225 minutes weekly).

This study found that, after statistically controlling for other variables, the time students spent on mathematics homework is highly associated with mathematics achievement and yet most of the students are assigned only a small amount of mathematics homework per week. Doing homework, as one of the ways students spend some of their time outside of school, may enhance their in-school academic performance (Lapointe et al., 1992). Therefore, eighth grade mathematics teachers should assign

an appropriate amount of homework to their students to further their understanding of concepts taught in class and thereby improve their achievement. Of course, the homework should be meaningful to students' mathematics learning rather than merely be "busywork" (DiGiulio, 1996; Gormas, 1996).

Provide More Advanced Mathematics Courses

Schools should provide all students with the opportunity to take advanced mathematics courses. Mathematics achievement in advanced areas cannot be acquired spontaneously by students from their surroundings outside school in the way that verbal skills can be (Moore & Smith, 1987). Participating in advanced mathematics courses in order to receive formal instruction in more complex and advanced mathematical concepts and processes is an important learning opportunity that contributes to student mathematics achievement.

In the present study, about half (50.4%) of the schools have less than one third of their eighth grade student body attending algebra class at least once a week. Only 13.2% of the schools have at least two thirds of their students attending algebra class at least once a week. Schools should provide their students with more sophisticated mathematical concepts and content in advanced areas. As one study revealed, "no evidence was found that requiring more students to take more advanced mathematics and science resulted in compromising the curricula of the courses experiencing the increased enrollments" (Porter, Kirst, Osthoff, Smithson, & Schneider, 1994, p. 6). Also, as argued above that low-achieving eighth grade students should be able to learn more than is typically demanded of them, schools are justified in replacing general math with more advanced mathematics courses, such as algebra, for the majority of the student body without content being "watered down." Furthermore, students in K-7 should be provided with adequate core mathematics knowledge and skills for their respective grades so that they can successfully continue to learn mathematics in eighth grade and beyond.

Increase Opportunity in Disadvantaged Areas

Schools, be they in an affluent or poor community, in urban, suburban, or rural area, be they public or private, should provide all students with access to a similar quantity and quality of opportunity to learn mathematics. Students' opportunity to learn mathematics should not be denied or restricted just because of the school they attend or the community in which they reside. Nevertheless, this study reveals that public schools with middle or low average student SES tended to provide less opportunity than their high average student SES counterparts.

Students should not be punished by being restricted to less opportunity to learn mathematics based on the fact that they grew up in a low SES family and community. Those areas where schools are providing less opportunity to learn to their students should receive priority assistance. Given current budget constraints, it is unlikely and might not be feasible to provide these schools with additional funds to improve

their situation. However, assistance should be provided to increase mathematical opportunity and resources without the necessity of extensive additional funding. For example, schools could encourage their teachers with adequate mathematics knowledge (usually reflected in the possessing of a mathematics degree) to help other teachers and to organize a mathematics club for students on a voluntary basis. Most importantly, schools could reduce the amount of high achievement groups, encourage teachers to provide high level of instruction commonly shared by high achievement groups, and offer most of their students the opportunity to take advanced mathematics courses.

Concluding Remarks

As reveals in this study, the distribution of OTL is not equal throughout different categories of schools. Public schools, and to a lesser degree Catholic schools, with low or middle average student SES provide less mathematical learning opportunity to their eighth graders than their counterparts with high average student SES. Students attending these schools are usually restricted in their opportunity to learn mathematics.

Overall, the expanded definition of OTL has accounted for more of the variance in student mathematics achievement than previous studies, but it is still not broad enough to account for a large proportion of the variance. Therefore, various kinds of further studies are needed because student achievement in general and mathematics achievement in particular remains a major concern to educators, the public, as well as the policymakers.

The NELS:88 has a nationwide sample of students and schools. Therefore, the findings of this study could be generalized to the nation. Based on the findings of the present study, policymakers may try to advocate that the education system provides their students with more opportunity to learn mathematics, such as to recruit, retrain and retain teachers with adequate mathematical knowledge, to encourage high content and level of instruction (including high level of instruction, coverage, and appropriate amount of homework), to provide more advanced mathematics courses, and, most importantly, to increase opportunities to learn in disadvantaged (or lower than average socioeconomic) areas.

Notes

¹No school OTL variable was specified in this model. However, the respective figures for school-level were 74.3% and 75.4%. Besides, they also conducted an alternative school-level analysis in which a school-level measure of mathematics achievement (the percent of eighth graders in a school that demonstrate high mathematics proficiency) was regressed with a school-level measure of students' access to early algebra (percentage of middle grades students who take a full year algebra) and other school and student population characteristics. The adjusted R square for the analysis was .43.

²This high achievement group variable represents the teacher's judgment about whether or not the class is high achievement in relation to the rest of the school. It serves as an indicator of the qualities commonly shared by high track classes, e.g., emphasizing teaching for understanding, high-order thinking, high/appropriate expectations, and challenging content. However, this variable includes more than opportunity to learn as students' abilities clearly influence their achievement level and whether they are in a high achievement group. To statistically control for students' abilities, students' prior performance is included as one of the control variables in the later analyses.

³OTL variables are aggregated to school-level and then grouped by the categories of school-level control variables. To facilitate the discussion, school-level control variables of Minority Concentration and School Average Student SES are divided into three categories (low, middle, and high). Unweighted cases ("Unweight Cases" for short) are added to each of the tables in order to provide information on the sample sizes of schools in each categories. Average School mathematics achievement ("School Math Ach") calculated from ALL students attending one school (excluding those who have missing values) is also added to the tables to provide a reference point in each category. Except for the unweighted cases, all of the variables are weighted both by the student- and school-level weights (BYQWT and BYADMWT) provided by the NELS:88 data set.

⁴Teachers' mathematical knowledge gained from professional development activities had no correlation with student mathematics achievement at all ($r = .0000$). (It was measured by teachers' report of time spent on in-service education in mathematics for the last 12 months) Instructional time, one of the two variables comprising the content exposure construct, was measured by teachers' report of number of hours per week class met regularly. It weakly, negatively correlated with student mathematics achievement ($r = -.0841$). Efforts have been made to detect the potential interaction of these two variables with other variables, such as teachers' academic degree, class type taught by the teacher, class type attended by students, and students' report of prior performance in mathematics since grade six. However, the investigator could not find one pattern that is able to explain the variance within these two variables of teachers' mathematical knowledge gained from professional development activities and instructional time. (There is no measure of teacher teaching effectiveness and students' prior

mathematics test scores in the NELS:88 data set.) Since both variables have been suggested in "An Expanded OTL Concept" section, the HLM analysis entered them into the proposed model and later deleted them in the revised model.

⁵His words were: "Sometimes we ask people to go." ... "Sometimes we feel a teacher needs to hear a presentation" then he or the building principal asked the teacher to attend it. Teachers never say no to him.

⁶The school-level control variables consist of six variables derived from four concepts, i.e. school sector, school minority concentration, community type, and school average student SES. Intended to reduce the number of the school-level control variables, this study has conducted a principal components analysis (J. Stevens, 1996). However, the results are not compelling enough to reduce the number of variables. Thus the six school-level variables are retained to cover the four concepts.

⁷In addition, schools located in affluent areas may also benefit from their community in other ways which are not included in the current study. For example, the schools may be able to pay their teachers better and therefore retain higher quality of teaching profession. Students' families may have higher expectations for their children and provide better support for their children's learning, students may have better role models, and the peer pressure may have less anti-intellectual atmosphere than those in low average student SES areas.

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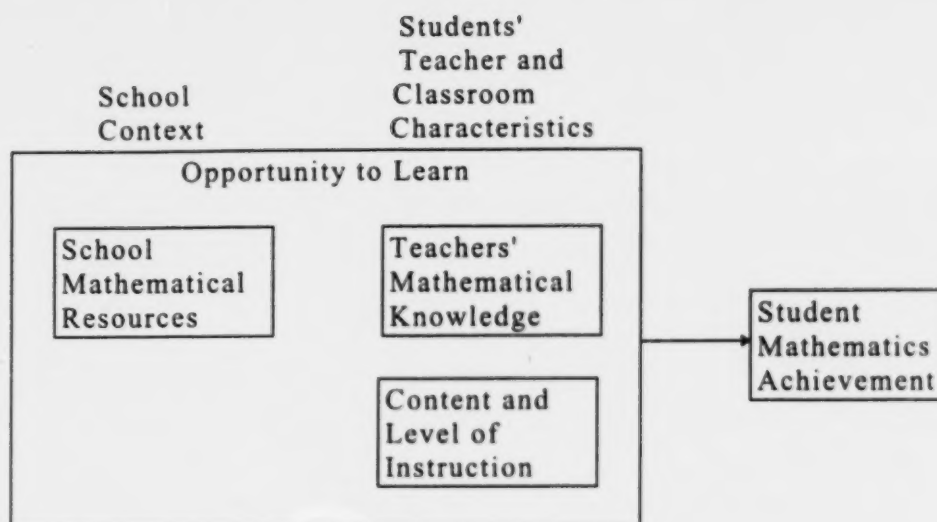


FIGURE 1. An expanded OTL concept

TABLE 1
The Distribution of OTL Across School Sector by School Average Student SES*

Sector/ Sch SES	Unweight Cases	Teacher Preservi %	Teacher Professi hr	Hi Achi Group %	Textbook Coverage %	Instruct Time hr	Homework min	Attendin Algebra %	Acess Algebra %	Math Club %	School Math Ach
Public	361	41.54	11.45	22.62	84.55	4.55	150.70	30.96	40.51	29.39	49.33
Low SES	143	36.68	11.76	19.26	82.99	4.59	155.74	26.53	39.05	21.31	47.17
Mid SES	132	40.56	9.73	21.88	85.92	4.61	142.86	33.35	36.22	33.43	50.83
Hi SES	86	61.17	14.05	36.35	87.26	4.30	149.39	41.83	55.07	49.82	53.88
Catholic	45	17.51	11.40	15.51	90.08	4.13	169.41	37.57	24.86	16.90	51.72
Low SES	5	.00	13.98	10.52	95.04	4.07	135.33	33.72	47.63	23.53	46.17
Mid SES	16	8.55	10.38	12.79	88.08	4.11	159.09	25.36	23.68	13.76	51.40
Hi SES	24	27.95	11.71	16.64	90.69	4.16	184.15	47.93	21.39	18.09	53.05
Othr Pri	40	30.24	8.54	49.92	89.45	4.38	138.92	53.97	28.30	17.69	55.30
Low SES	0										
Mid SES	1	.00	3.50	.00	85.00	5.00	150.00	.00	.00	.00	52.42
Hi SES	39	33.54	9.09	55.37	89.94	4.32	137.71	59.87	31.39	19.62	55.62
Low SES	148	34.49	11.89	18.74	83.71	4.56	154.52	26.96	39.56	21.44	47.11
Mid SES	149	29.98	9.75	18.58	86.53	4.47	147.84	30.05	31.50	26.69	51.05
Hi SES	149	40.48	11.76	33.86	89.35	4.25	159.82	49.21	35.29	29.01	54.04

*Except for the "Unweight Cases", variables are weighted both by the student- and school-level weights.

TABLE 2
The Distribution of OTL Across Urbanicity of School Location by Minority Concentration*

Urbanic/ Min Conc	Unweight Cases	Teacher Preservi %	Teacher Professi hr	Hi Achi Group %	Textbook Coverage %	Instruct Time hr	Homework min	Attendin Algebra %	Acess Algebra %	Math Club %	School Math Ach
Urban	130	32.63	12.38	27.36	86.49	4.38	150.78	39.73	31.76	34.80	50.14
Low Min	54	35.97	9.58	29.94	87.43	4.23	136.18	40.93	32.62	25.67	52.62
Mid Min	31	29.10	10.80	36.98	88.08	4.35	150.01	38.31	25.86	49.44	48.73
Hi Min	35	25.82	22.05	11.27	82.28	4.87	194.85	37.39	34.49	49.08	44.00
Suburb	182	32.02	11.37	21.43	87.23	4.34	165.19	39.08	35.61	22.79	51.03
Low Min	153	35.07	10.96	23.57	86.68	4.34	164.52	40.30	37.59	24.16	51.81
Mid Min	16	13.49	13.10	12.57	90.62	4.09	174.60	27.81	6.76	5.67	48.46
Hi Min	13	20.56	14.18	7.01	89.21	4.66	159.45	40.44	53.92	31.10	44.73
Rural	134	39.05	10.18	22.25	85.29	4.57	145.46	27.04	38.44	21.63	50.08
Low Min	116	36.53	10.54	22.69	85.68	4.54	144.56	26.89	40.83	22.35	50.80
Mid Min	11	64.17	4.55	36.30	88.15	4.87	188.84	48.65	39.95	35.99	45.68
Hi Min	7	60.04	8.75	5.18	77.11	4.89	128.01	13.59	.00	.00	42.03
Low Min	333	35.87	10.51	24.42	86.40	4.40	150.53	34.68	38.02	23.68	51.53
Mid Min	58	29.14	10.62	27.91	89.03	4.34	165.42	36.16	21.17	31.16	48.13
Hi Min	55	32.31	17.15	8.87	82.77	4.83	171.23	32.73	31.28	33.65	43.73

*Except for the "Unweight Cases", variables are weighted both by the student- and school-level weights.

TABLE 3
HLM Analysis Results of Student-Level Models

Variables	Proposed Model				Revised Model			
	β	S β	SE	t-ratio*	β	S β	SE	t-ratio*
<u>Control Variables</u>								
Gender	.761	.055	.024	31.402	.758	.054	.024	31.275
Race/Ethnicity	2.296	.138	.044	51.942	2.297	.138	.044	51.930
Family SES	2.053	.221	.021	99.600	2.051	.220	.021	99.505
Prior Performance	2.923	.423	.013	226.062	2.924	.423	.013	226.018
<u>Teachers' Math Knowledge</u>								
Mathematics Degree	.278	.020	.047	5.857	.362	.026	.047	7.692
Professional Develop.	-.054	-.008	.020	-2.731				
<u>Content and Instruction</u>								
High Achievement Group	8.758	.552	.037	235.952	8.752	.552	.037	235.660
Textbook Coverage	.333	.061	.017	19.223	.315	.057	.017	18.180
Instructional Time	-.777	-.079	.039	-19.804				
Weekly Homework	.146	.174	.003	54.020	.140	.167	.003	52.295

*The t-ratios in this table are all significant at $p < .001$ level, except for the Professional Development one, which is at $p < .01$ level.

β : Unstandardized regression weight.

S β : Standardized regression weight.

TABLE 4
 ILM Analysis Results of School-Level Models

Variables	Proposed Model				Revised Model			
	γ	S γ	SE	t-ratio	γ	S γ	SE	t-ratio
Control Variables								
Catholic School	-.158	-.027	.513	-.309	.151	.026	.507	.299
Other Private School	.527	.063	.692	.761	.717	.086	.684	1.049
Minority Concentration	-.014	-.161	.006	-2.166 *	-.016	-.186	.006	-2.585 *
Suburban School	-.007	-.001	.393	-.017	-.007	-.001	.388	-.018
Rural School	.633	.127	.456	1.389	.606	.122	.450	1.347
School SES	5.321	.958	.479	11.106 ***	5.421	.984	.473	11.463 ***
School Math Resources								
Algebra Classes	-.005	-.050	.007	-.685	-.004	-.041	.007	-.596
School Calculators	.006	.107	.003	1.705	.006	.108	.003	1.645
Mathematics Club	-.065	-.012	.309	-.210	-.127	-.023	.305	-.417
Without School SES Model								
Variables	γ	S γ	SE	t-ratio				
Control Variables								
Catholic School	1.289	.191	.567	2.272 *				
Other Private School	4.292	.439	.692	6.200 ***				
Minority Concentration	-.052	-.516	.006	-8.459 ***				
Suburban School	.311	.056	.441	.706				
Rural School	-.563	-.097	.500	-1.125				
School SES								
School Math Resources								
Algebra Classes	.025	.218	.008	3.177 **				
School Calculators	.006	.093	.004	1.554				
Mathematics Club	.451	.070	.343	1.313				

* $p < .05$, ** $p < .01$, *** $p < .001$

γ : Unstandardized regression weight.

S γ : Standardized regression weight.

TABLE 5
HLM Results of Variance Explained at Student-Level in Revised Model*

	Fully Uncondi. Model [^]	Control Variables Only	Control Plus Teacher Math Knowledge	Control Plus Content and Level of Instruc.	Control Plus All
Student-Level Variance	81.56	60.51	60.31	49.00	48.99
Variance Explained		25.81%	26.04%	39.92%	39.93%

* School-Level variables are not entered in these models.

[^] The "Fully Uncondi. Model" (Fully Unconditional Model) contains no variable.

TABLE 6
HLM Results of Variance Explained at School-Level in Revised Model*

	Fully Uncondi. Model [^]	Control Variables Only	Control Plus School Math Resource
School-Level Variance	16.50	5.92	5.88
Variance Explained		64.12%	64.36%

* Student-Level variables are entered in the last three models.

[^] The "Fully Uncondi. Model" (Fully Unconditional Model) contains no variable.